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ESTIMATION OF FRICTION OF SURFACE WINDS IN THE AUGUST 1949, FLORIDA HURRICANE

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ABSTRACT

The equations of motion are adapted for use in computing the tangential and normal components of friction in a model hurricane. Values of the components of friction are estimated from surface data of the August 1949 Florida hurricane. The computations are based upon mean pressure and wind profiles describing the front and rear portions of the storm, within 28 to 70 miles from the center, as it moved through the Lake Okeechobee region. The tangential component of friction appears to be greater than the normal component in the region to the rear of the center, while in advance of the center the opposite is generally observed. All friction curves show a continuous increase of friction with wind speed.

INTRODUCTION

In cooperation with the Corps of Engineers, U. S. Army, the Hydrometerological Section of the United States Weather Bureau has been investigating surface winds of hurricanes, with particular interest in the region of Lake Okeechobee, Fla. One phase of the investigation was concerned with the forces resisting surface winds. Although the frictional forces elude exact measurement, their importance to the behavior of surface winds is readily apparent. A familiar approach to the problem of computing friction is through the theory of the mixing length, assuming that the mixing length can be determined and that the velocity distribution, especially along the vertical, is known. This method is discussed in detail by Rossby and Montgomery [1]. Since the hurricane data available to the Section were marked by an absence of upper air observations, further consideration of the mixing-length theory was necessarily eliminated. The equations of motion, therefore, were developed in a form suitable for use with hurricane surface data to compute friction. Their development and the results subsequently obtained are presented in this paper.

THE EQUATIONS OF MOTION

The equations of horizontal motion may be written

$$\frac{dv_s}{dt}=-\alpha\frac{\partial p}{\partial s}-F_s$$
$$\frac{v_s^2}{r_s}+fv_s=-\alpha\frac{\partial p}{\partial n}-F_n \tag{1}$$

where v_s is the speed along the path, s; r_s, radius of curvature of s, is positive for cyclonic curvature; t is time; f the Coriolis parameter; α the specific volume; p the pressure; n, a normal to s, is directed opposite to the Coriolis force; and F_s and F_n are the components of friction, tangent and normal, respectively, to the path. For convenience in a study of friction the above equations are rewritten

$$F_s=-\alpha\frac{\partial p}{\partial s}-\frac{dv_s}{dt}$$
$$F_n=-\alpha\frac{\partial p}{\partial n}-\frac{v_s^2}{r_s}-fv_s \tag{2}$$

Friction is defined here as the sum of all forces which are due to friction and viscosity. In addition to the friction arising from flow over a rough ground surface, it includes the effects of lateral eddy viscous forces, whether accelerative or retardative, imposed by adjacent air currents.

METHOD OF CALCULATING THE FRICTION IN A MODEL HURRICANE

The solution of equations (2) for a model hurricane, characterized by a steady state of mean flow, circular symmetry, and constant latitude, is given in this section.

It is seen from the working model, shown in figure 1, that

$$\begin{aligned} ds &= -dr/\sin\theta \\ dn &= -dr/\cos\theta \end{aligned} \tag{3}$$

where r, the distance from the center, is directed outward, and θ is the deflection angle of the wind as measured from the normal of r to the tangent of the trajectory s. Since the mean flow has been assumed to be steady,

$$\frac{dv_s}{dt} = v_s \frac{\partial v_s}{\partial s} \tag{4}$$

In accordance with Yates [2], the radius of curvature of a spiral may be written

$$r_s = r\frac{dr}{dq} \tag{5}$$

where q, a normal to the tangent of the spiral, is drawn from the origin as shown in figure 1. From figure 1 it is seen that

$$q/r = \cos\theta \tag{6}$$

Differentiating equation (6) with respect to r and substituting the result into equation (5) gives

$$r_s = \frac{1}{\dfrac{\cos\theta}{r} - \sin\theta\dfrac{d\theta}{dr}} \tag{7}$$

Substitution from equations (3), (4), and (7) into equations (2) gives

$$\begin{aligned} F_s &= \left(\alpha\frac{\partial p}{\partial r} + v_s\frac{\partial v_s}{\partial r}\right)\sin\theta \\ F_n &= \left(\alpha\frac{\partial p}{\partial r}\right)\cos\theta - v_s^2\left(\frac{\cos\theta}{r} - \sin\theta\frac{\partial\theta}{\partial r}\right) - fv_s \end{aligned} \tag{8}$$

The friction at a selected distance from the center of the model hurricane is computed by using equations (8) with values obtained from three curves showing the respective relations of pressure p, wind speed v_s, and deflection angle θ, to distance r from the center. Representative values of virtual temperature and latitude must of course be selected for evaluating the specific volume and Coriolis parameter.

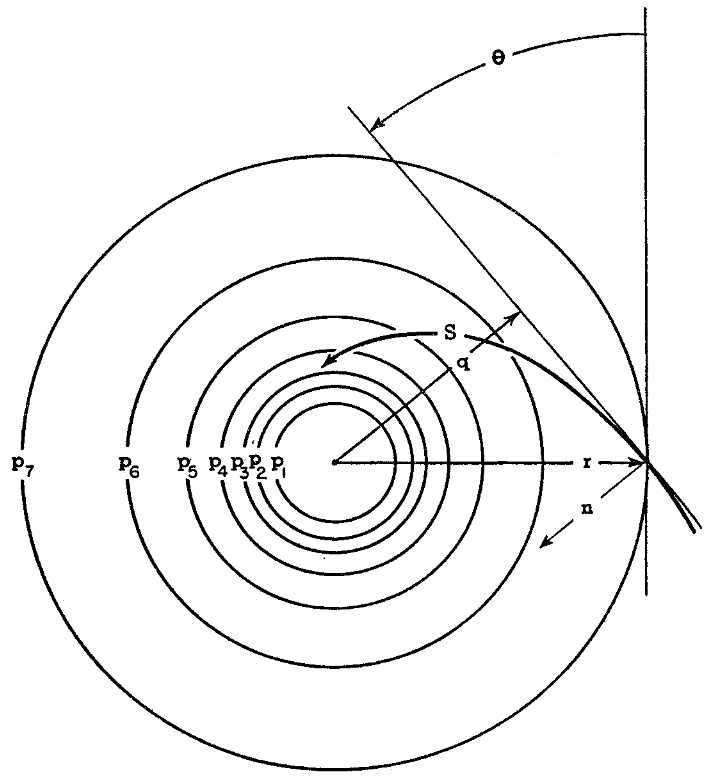

FIGURE 1.—Hurricane model.
p_i = Value of a circular isobar.
r = Distance from the center, directed positive outward.
s = Path of air particle, directed positive in the direction of motion.
θ = Deflection angle of the wind across a circular isobar.
n = Normal to s, directed opposite to the Coriolis force.
q = A normal to tangent of path, drawn from center.

INSTRUMENTATION

The Corps of Engineers maintains meteorological observation stations at seven hurricane gates on the shore of Lake Okeechobee and three wind-recording stations on steel-girder pylons within the lake. The station locations are shown in figure 2. Esterline-Angus multiple pen recorder traces of the wind speed and direction were available for the lake stations. Dines anemometers at the hurricane gates provided continuous traces of wind speeds and directions except for Hurricane Gate No. 3 where wind speeds only were available. The hurricane-gate stations also provided barograph traces. The heights of anemometers are given below in table 1. The water surface is about 14 ft. msl and the land surface 14 to 16 ft. msl.

TABLE 1.—*Anemometer height at observation stations at Lake Okeechobee*

Station	Anemometer height (ft. msl)
Hurricane Gate No. 1	58
Hurricane Gate No. 2	55
Hurricane Gate No. 3	56.5
Hurricane Gate No. 4	55
Hurricane Gate No. 5	55
Hurricane Gate No. 6	55
Port Mayaca	56
Lake Station No. 12	46.5
Lake Station No. 14	48.5
Lake Station No. 16	47.5

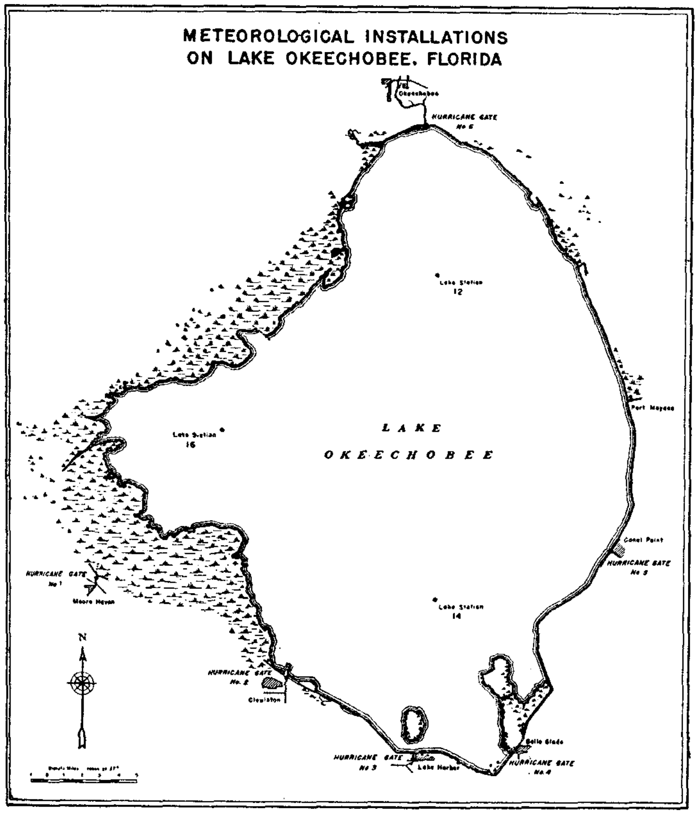

FIGURE 2.—Map of Lake Okeechobee area showing location of observation stations.

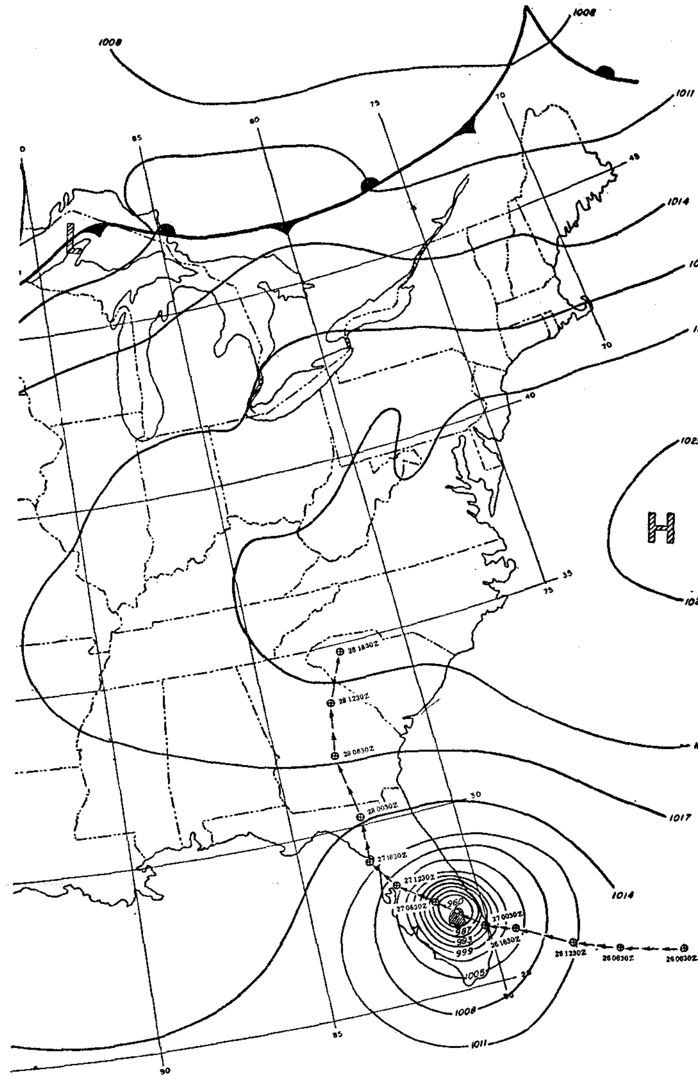

FIGURE 3.—Section of surface chart for 0330 GMT, August 27, 1949, showing current position and path of hurricane.

ESTIMATION OF FRICTION IN THE AUGUST 1949 FLORIDA HURRICANE

The method presented for calculating the friction was derived by assuming a symmetrical hurricane with steady mean flow and no variation in latitude. These assumptions were convenient in order to arrive at an exact mathematical expression for the friction. Although the assumptions may be reasonably well satisfied in nature by a quasi-stationary hurricane showing little change in intensity, no hurricane is expected to satisfy the assumptions perfectly. Because an ideal system is lacking, computed values of friction must properly be regarded as estimates.

Although the data of this hurricane were the best available to the Hydrometeorological Section, they were not entirely satisfactory. Insufficient synoptic data led to the use of observational histories of the 10 stations at Lake Okeechobee. The basic data were extracted at 10-minute intervals from recording charts. Because of the fluctuations in wind speed and direction, each wind datum was extracted as a 10-minute average value. The deflection angles of the wind were measured with reference to circles having their origin at the rotation center. The locations of the stations relative to the hurricane center, whose path is traced in figure 3, were determined for successive 10-minute positions of the center for the times corresponding to those of the data extracted from the station recording charts. The station-location data used with the station records determined the wind speeds, deflection angles, and pressures for positions in the hurricane. Thus, each station furnished data for several positions in the front and rear portions of the storm. The data used for the front were observed during a 5-hour period as the storm moved 85 to 90 miles northwestward toward Lake Okeechobee. The central pressure of the storm remained nearly constant within the range 28.15 to 28.20 inches. The data used for the rear were observed during a 5-hour period in which the storm progressed about 65 miles in the region beyond the Lake and the central pressure rose from about 28.20 to 28.40 inches.

The data observed at each 10-minute position of the storm are plotted in figures 4–6. For any given station these plotted points will of course be spaced farther apart the faster the movement of the storm center and the closer the station lies to the path of the center. Mean curves based only upon these plotted points would therefore be biased in favor of those stations which, because of their locations, provided a greater number of observations

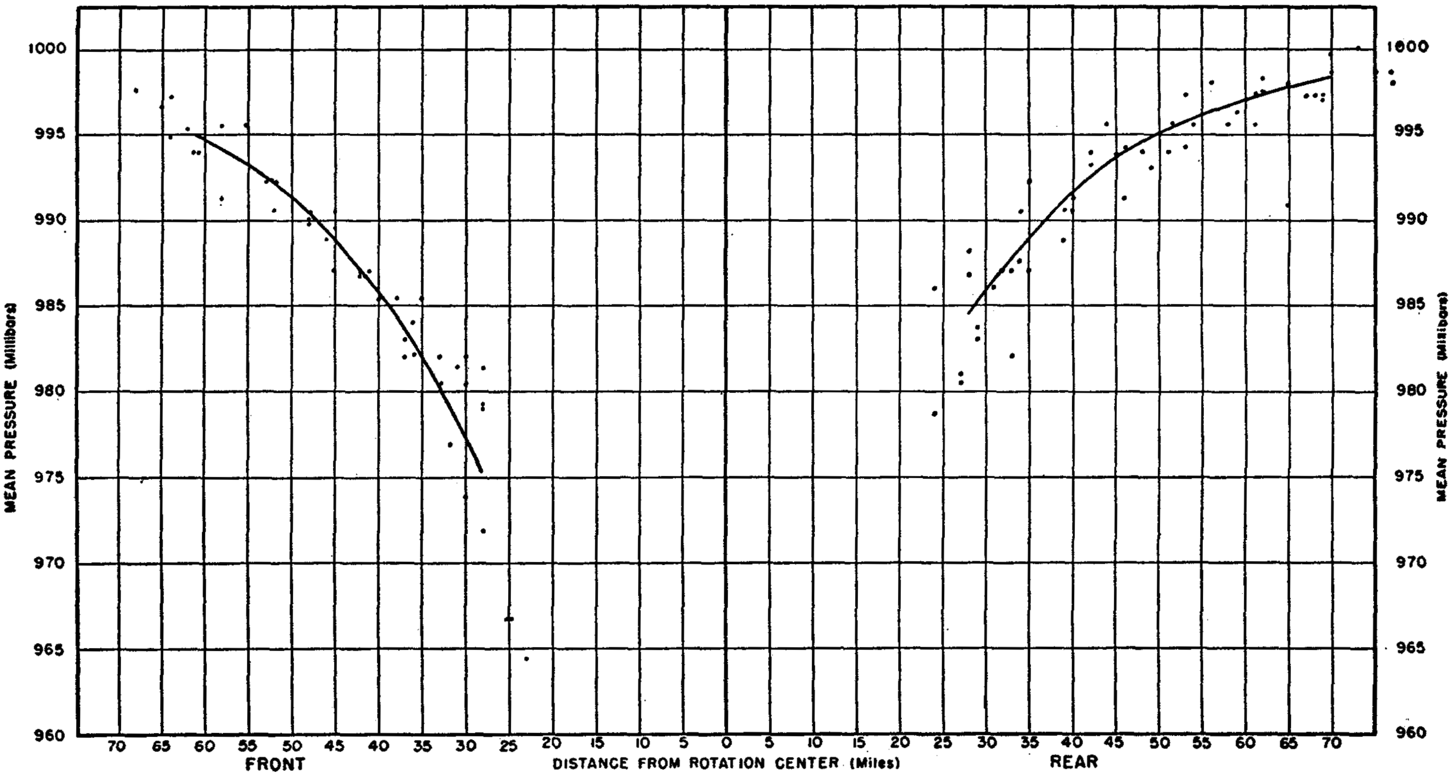

FIGURE 4.—Mean pressure profiles for front and rear areas of hurricane of August 1949.

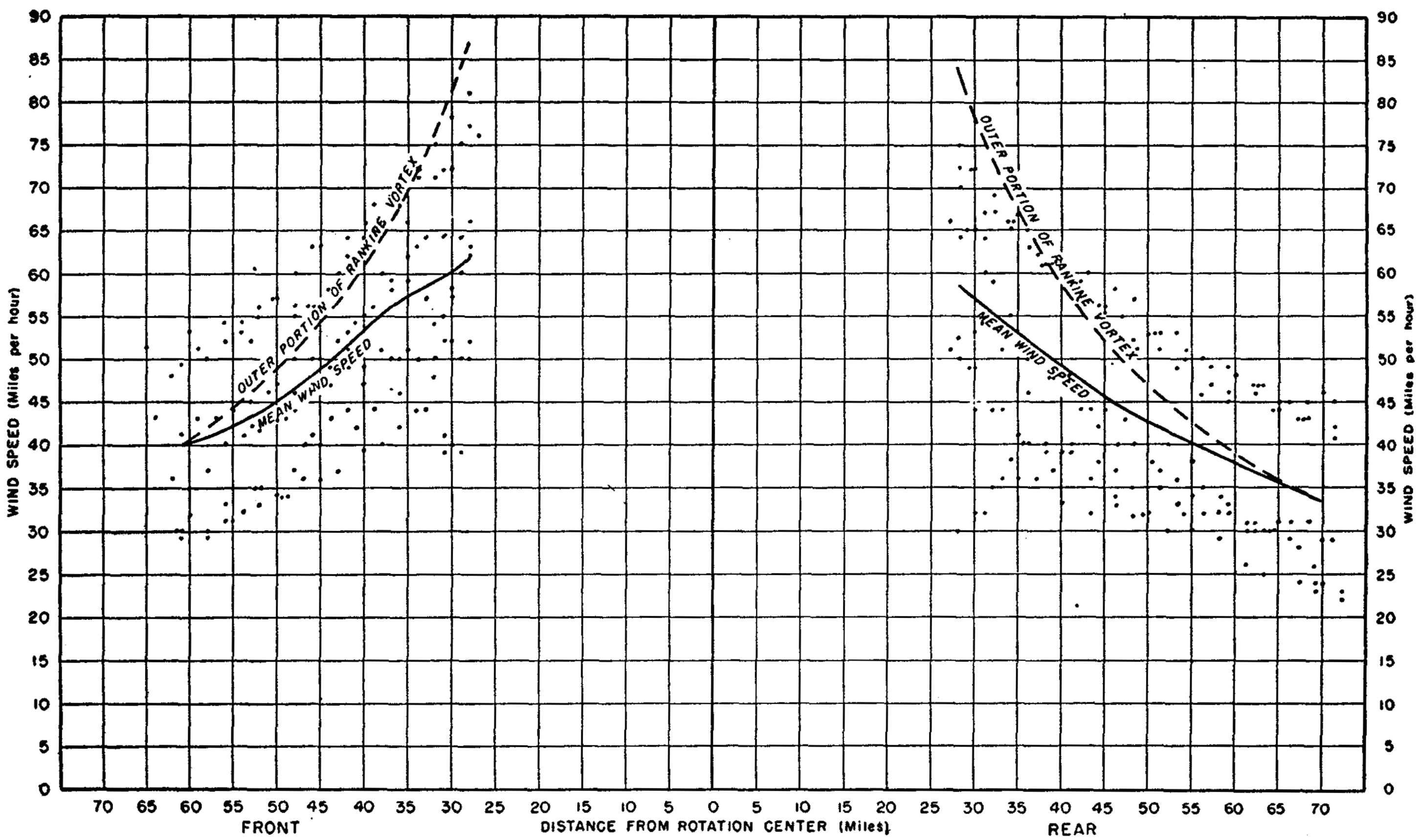

FIGURE 5.—Mean wind speed profiles, each compared with a Rankine Vortex, for front and rear areas of hurricane of August 1949.

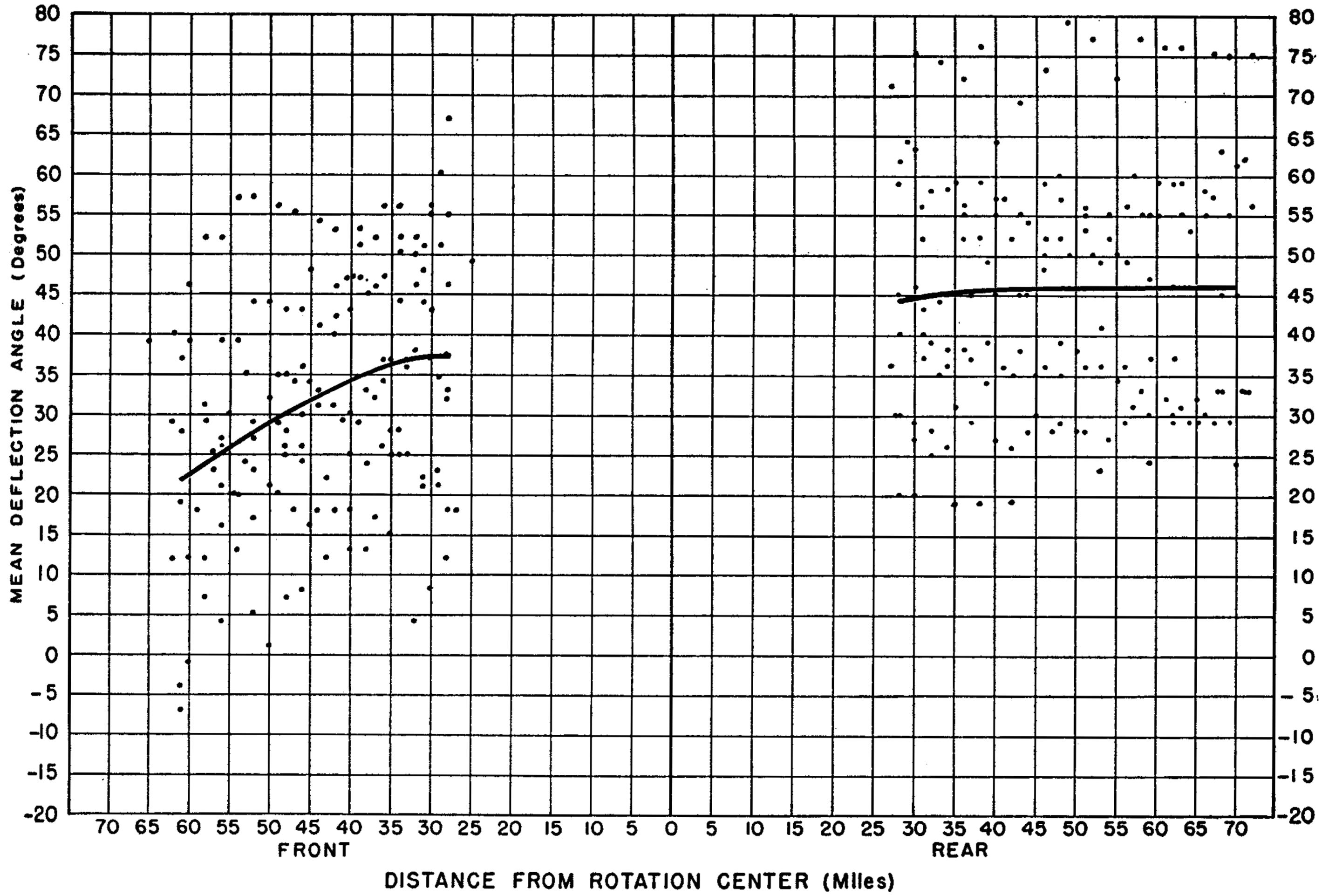

FIGURE 6.—Mean deflection angle profiles for front and rear areas of hurricane of August 1949.

within the part of the storm represented by the curves. With the purpose of allowing all stations to have equal influence on the curves, interpolations were made from a tabulation of the 10-minute observational data for each station in order that every station might be represented by data, observed or interpolated, at every mile distance from the center. During the 10-minute time intervals between observed data, changes were very small and linear interpolations within these small increments appear to be highly satisfactory. The complete tabulation of observed and interpolated data was used to compute arithmetic averages of pressure, wind speed, and deflection angle for every mile distance from the center. Thus, equal weight was given to all stations in determining the mean profiles shown in figures 4–6. Although the data of the several stations on which each of these curves is based were observed over a 5-hour period, the average times of the data corresponding to the points on a curve all fall within a period of only about 2½ hours. Since the center of the hurricane passed within 28 miles of every station at Lake Okeechobee it was possible to construct the curves for the region outside the 28-mile radius. Continuation of these mean curves within the 28-mile radius was not permitted in view of a bias introduced by the decrease in the number of observing stations with nearness to the center. Extension of the curves beyond 60 or 70 miles from the center was not possible because of the uncertain position of the center when it was located out at sea and later because of the filling which occurred after the center had moved beyond Lake Okeechobee.

It can be seen from figure 3 that the storm center moved over the northeastern part of the Lake and, hence, most of the data were from the left side of the storm. At all stations the winds were influenced by both land and water in proportions which varied with each change in the relationship of the wind pattern to the land-water distribution. The mixed influence of land and water in various proportions contributed to the scatter of observed wind values. In figure 5 the separation of points plotted for the rear of the storm into two groups according to wind speed is due to relatively large differences in the proportional influence of land and water. Since the winds were under a mixed influence, it was not possible to make a comparative analysis of pure-water and pure-land winds. By using the mean curves, figures 4–6, with equations (8), the mean friction was estimated at a sufficient number of distances from the rotation center to define the friction curves shown in figure 7. In the computations the air was considered saturated at 75° F. and the latitude was taken as 27° N.

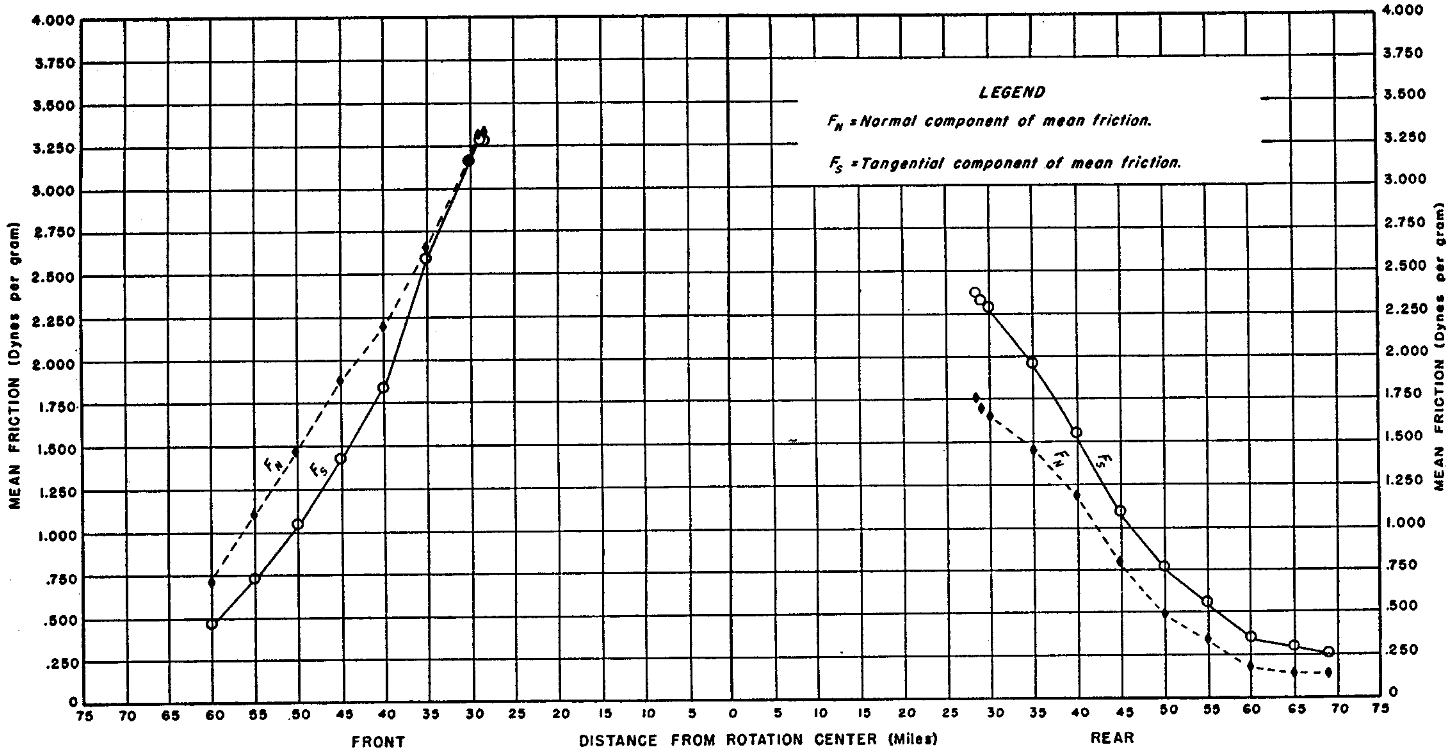

FIGURE 7.—Distribution of mean friction in front and rear areas of hurricane of August 1949.

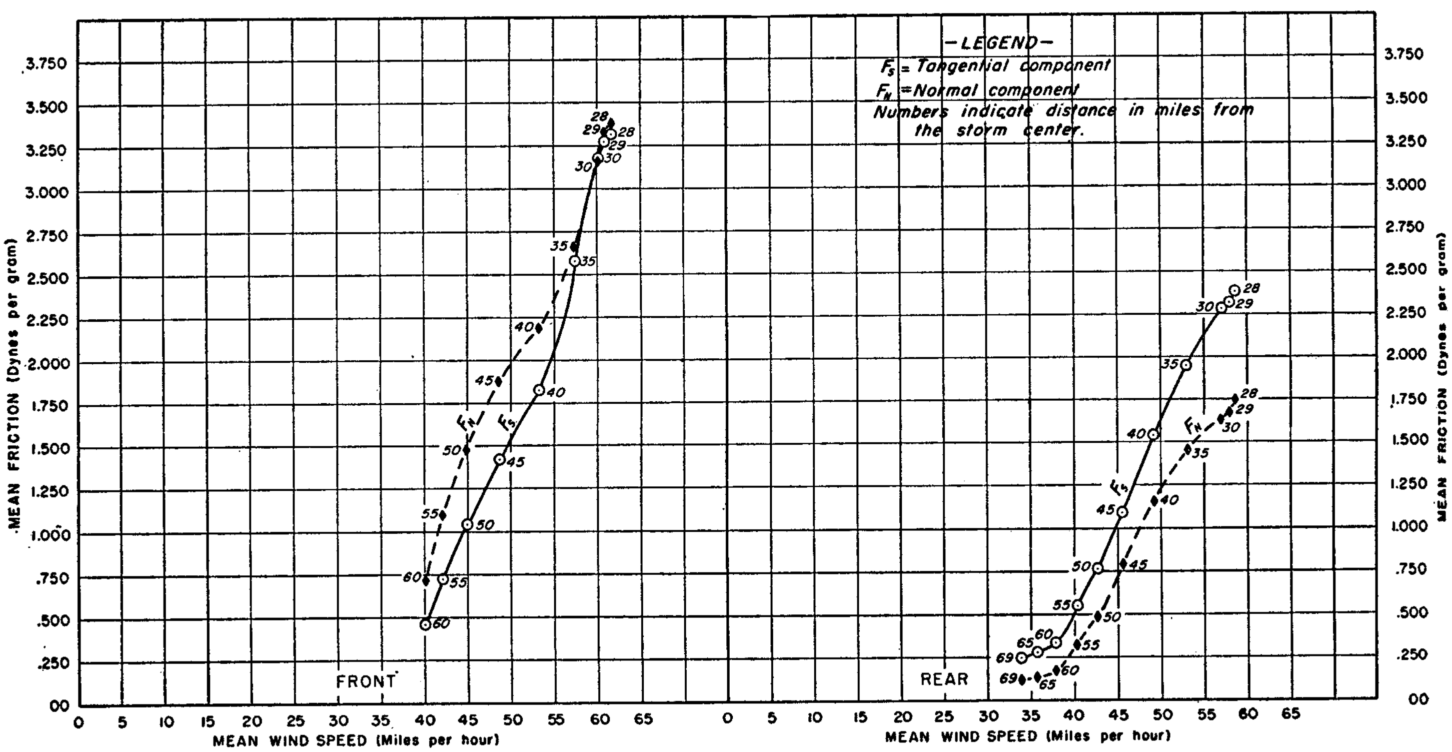

FIGURE 8.—Relations of components of mean friction to mean wind speed for front and rear areas of hurricane of August 1949.

DISCUSSION

The dearth of data from the inner regions of the storm excluded the most interesting zones from this study. Only six stations experienced the hurricane within the radius of maximum winds. On the basis of their reports the mean values of wind speed and deflection angle in front of the storm center were greatest at a radius of 25 miles. In the rear of the center the greatest mean value of wind speed was indicated at a radius of 22 miles. Unlike the forward portion of the storm, the rear portion was not marked by a radius having a peak value of the mean deflection angles. The mean deflection angles in the rear showed a continuous increase with distance from the center until a constant value was reached at the 45-mile radius. In each part of figure 5 a graph of the wind-speed distribution in the outer portion of a Rankine Vortex is included for comparison.

The friction curves in figure 7 show the variations of the tangential and normal components of mean friction with respect to distance from the rotation center of the hurricane. It would be expected that the tangential component is greater than the normal component. This was, in fact, found to be the case in friction computations by Horiguti [3], when the equations of motion were applied to mean data of the Okinawa typhoon of August 1924. While Horiguti's data were of the region of the typhoon between 62 and 435 miles from the center, the data leading to the analysis given here were from the region within 70 miles of a storm center and were treated separately for the front and rear portions of the storm. In the present analysis, the tangential component again was found to be greater than the normal component in the region to the rear of the center. But in front of the center, as shown in figure 7, the normal component generally appears to be the greater.

The relations of the tangential and normal components of mean friction to mean wind speed are presented in figure 8. Each curve shows a continuous increase of friction with wind speed. The number beside each plotted point indicates the distance from the storm center.

Errors introduced in the computations by assuming horizontal flow, constant virtual temperature of surface air, and a mean latitude are considered negligible. The subjectivity in measuring slopes of the basic curves is responsible for some error in the results, but careful measurements have kept this error small.

The representation of the front and rear portions of the hurricane as parts of symmetrical systems in steady mean states, by mean curves, served in arriving at estimates of mean friction. Although the mean curves have a virtue in eliminating local effects imposed upon the representative flow, there may be an objection to comprehensive means which obscure large-scale dynamic influences characteristic of different sectors of the storm. In computations of mean friction the results can vary with the manner of defining the mean flow. Hence, there is no criterion for judging errors incurred through the use of mean flows.

Without observational data through the vertical, there is no way for direct verification of the results. With the aid of upper air data the method presented in this paper may provide a means of verifying the standard theoretical formulae for computing friction.

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